

Let's take a constructive approach to get a better idea of how the spaces (manifolds) and Hamiltonians H relate.

The simplest space would be a 2D plane (q, p) , where q denotes position and p denotes momentum. Ignoring constants, this is the phase space of a free point particle traveling on a line $H(q, p) = p^2$, and the harmonic oscillator $H(q, p) = q^2 + p^2$.

In these simple contexts, what does $dp \wedge dq = 0$ mean?

A more interesting question might be: why is the symplectic Euler discretization stable? And why is it slightly skewed?