

Detuning by adding a little frequency is the same as subtracting a little period. With $p = \frac{1}{f}$ we have

$$\frac{df}{f} = \frac{d\frac{1}{p}}{\frac{1}{p}} = -\frac{dp}{p}.$$