One of the things I never really understood properly, is

$$D\log(x) = \frac{1}{x}.$$

This essentially comes from two properties: derivatives of inverses are related through multiplicative inverses, and the exponential function is an eigenfunction of the derivative operator.

The expression of the derivative of the inverse Df^{-1} can be obtained from the chain rule as

$$D(f \circ f^{-1}) = [(Df) \circ f^{-1}]Df^{-1}.$$

Since this also expresses the derivative of the identity function, we have

$$Df^{-1} = \frac{1}{(Df) \circ f^{-1}}.$$

This relation is obvious for a linear function, e.g. $y = ax \Rightarrow x = a^{-1}y$. Also, following the composition of the 3 operations f^{-1}, Df and $x \to 1/x$ on a plot makes it fairly obvious what is going on is just obtaining the tangent line and expressing it in the proper coordinate system.

The expression for $D\log$ follows straight from the rule of the derivative of the inverse of $\exp$.

$$D\log(x) = \frac{1}{\exp[\log(x)]} = \frac{1}{x}.$$