Basic idea is to provide a scale that is mostly log–like, but can express the extremities 0 and .

How to map this time scale $T = [0, \infty]$ to a control parameter range $C = [0, 1]$ in a meaningful way? A mapping that satisfies the interval boundaries is

$$c(t) = \frac{t}{a+t}.$$

This maps $c(0) = 0$ and $c(\infty) = 1$. The inverse is

$$t(c) = \frac{ac}{1-c}.$$

The parameter a can be determined by constraining the middle of the scale $t(1/2) = a$. Another important parameter is how fast the dial will move to 0 and ∞. Both are extremes that are part of T, but for say 50% of the T range we'd like to have decay rates that change mostly exponential in c.

To expose the symmetry in $t(c)$, let's introduce a change of variable $d = 2c - 1$. This gives

$$t(d) = a\frac{1+d}{1-d}.$$

This scale is logarithmically symmetric around a or $t(d)/a = (t(-d)/a)^{-1}$, meaning that in the middle range it behaves mostly exponential, while tending to 0 and ∞ in the two extremes[1].

The slope of $t(d)$, relative to a is fixed. At $d = 0$, the function approximates $a \exp(2d)$. For mapping meaningful parameters, this might be a bit too flat. Successive squaring of $t_0(d) = (1 + d)/(1 - d)$ can solve this. For n squarings we have $\exp(2nd)$. The curve then becomes

$$t_n(d) = at_0(d)^{2^n}.$$

In practice it seems that a single squaring works good. This gives a reasonably flat log response for 3 decades, about 70% of the scale, leaving the rest for the extreme range. If necessary, the extremities can be avoided by prescaling d. With one squaring, using 0.99 limits the output range to about 9 decades.

[1]This hints at the input–scaled variant $\frac{1+ax}{1-ax}$ being a good approximation for e^x, which is the case when $a = \frac{1}{2}$.

Mapping this to pole radius requires an extra step. Let's use the natural $1/e$ decay to relate decay time t (measured in samples) to pole p as $p^t = 1/e$ or

$$p = \exp(-\frac{1}{t}).$$

This approximation needs to be accurate for $t \gg 1$, and extend correctly to $p = 0$ at $t = 0$. The first degree Taylor expansion is $1 - 1/t$. Modifying this slightly to give

$$p' = 1 - \frac{1}{t+1}$$

yields the wanted behavior at $t = 0$ without changing the large t behavior too much. For numerical reasons the update equations will use the positive quantity

$$q' = \frac{1}{t+1},$$

where $p' = 1 - q'$. Composing the two mappings gives

$$q_n(d) = \frac{1}{1 + a(\frac{1+d}{1-d})^n}.$$

where a gives the mid–scale time constant in samples.

I'm using $n = 2$, but some knob twiddling makes me think that maybe $n = 1$ is better. What is important is to get the mid–scale value correct. I.e. what is a prototypical note's attack and decay rate?

To find the warping