The state space form of the harmonic oscillator consists of the following system of differential equations

$$\begin{aligned} \dot{q}_1(t) &= -\omega q_2(t) \\ \dot{q}_2(t) &= \omega q_1(t) \end{aligned}$$

This can be solved by Laplace-transforming the differential equations to a set of equations

$$\begin{aligned} sQ_1(s) - q_1(0) &= -\omega Q_2(s) \\ sQ_2(s) - q_2(0) &= \omega Q_1(s) \end{aligned}$$

Solving this for $Q_1(s), Q_2(s)$ gives a solution in the Laplace domain, which can be converted back to the time domain. The solutions looks like

$$Q_i(s) = \frac{n_i(s)}{s^2 + \omega^2}$$

where the numerators $n_i(q)$ are first order polynomials in terms of the initial conditions $q_i(0)$. The interesting part is the denominator $s^2 + \omega^2 = (s - j\omega)(s + j\omega)$ which determines the *shape* of the result. Splitting the $Q_i(s)$ in partial fractions then gives the sum of first order rational functions proportional to $(s - j\omega)^{-1}$ and $(s + j\omega)^{-1}$, which are Laplace transforms of $e^{j\omega t}$ and $e^{-j\omega t}$ respectively.

The denominator can also be obtained as $\det(Is - A)$ where A is the system matrix

$$A = \begin{bmatrix} 0 & -\omega \\ \omega & 0 \end{bmatrix}.$$