To use the N-R for function evaluation $f(x)$ requires an extra step, which is to construct a function F such that $F(f(x)) = 0$. Is there a systematic way to do this?

As an example, take $f(x) = 1/\sqrt{x}$. Construct a function $F(q)$ such that $F(1/\sqrt{x}) = 0$. The straightforward approach would give something like

$$F_1(q) = q - 1/\sqrt{x}.$$

Computing the N-R update from F_1 is rather useless

$$u_1(q) = q - \frac{q - 1/\sqrt{x}}{1} = 1/\sqrt{x}.$$

The trick seems to be to take this trivially obtained but useless F_1 and change it in a way that turns the dependency on x into elementary functions, without removing the zero of F_1 we are interested in. The following two steps seem appropriate: remove the square root

$$F_2(q) = (q - 1/\sqrt{x})(q + 1/\sqrt{x}) = q^2 - 1/x$$

and remove the division operation

$$F_3(q) = xq^2 - 1$$

Which of the two is best? There is no way to decide until computing the N-R update steps. It turns out that both lead to the same function

$$u_2(q) = u_3(q) = \frac{1}{2}(q + \frac{1}{xq}).$$

Now $u_2(q)$ still contains a division, which is not optimal. There are other ways. In [1] the function $F_4(q) = 1/q^2 - x$ is used, which leads to

$$u_4(q) = \frac{1}{2}(3q - xq^3).$$

The heuristic to distill from this is that if there is a division involved, it might be best to express $F(q)$ in terms of $1/q$. The derivative will have a decreasing negative power. The negative power is then cancelled by $F'(q)$ appearing in a denominator in the update formula.

That trick should then also work for evaluating $f(x) = 1/x$. Indeed, with $F_5(q) = 1/q - x$ we get $u_5(q) = q(2 - xq)$.