Reading "Finite Difference Equations" by Levy and Lessman. Interesting book. Opens my eyes about a lot of things related to the continuous and discrete cases. The one that struck me today is

$$2^x = (1+1)^x = \sum_n \binom{x}{n} = \sum_n \frac{x^{\underline{n}}}{n!}.$$

Here $x^{\underline{n}} = x(x-1)\dots(x-n+1)$, the falling powers of x. This is a notation due to Knuth; the book uses $(x)_n$ which I find less clear. The expression akin to the Taylor expansion is

$$f(x_0 + xh) = \sum_n \frac{x^{\underline{n}}}{n!} \Delta^n f(x_0),$$

where $\Delta^n f(x_0)$ is the n times iterated difference operation $\Delta f(x) = f(x+h) - f(x)$. The sum is finite when f is a polynomial, and is otherwize defined if f is analytic. The relationships between the difference operator Δ, the shift operator $E = 1 + \Delta$, and the differentiation D are expressed as

$$E = e^{hD}$$

and

$$\log(1+\Delta) = \log E = hD,$$

where the exponential and logarithm signify the usual power series of the operators.